Information Aggregation Mechanisms

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Abstract

In this short note, we describe an information aggregation mechanism that can be used by players before playing a game of strategic complementarities under incomplete information. In such a game, players may have an incentive to share overly optimistic information with others, thereby inducing them to take higher actions. In this mechanism, players trade a token before playing the game. Players who wish to communicate good news must purchase this worthless token and burn resources. The note shows that players only need to observe the market-clearing price that arises from the token trades to aggregate their private information. Each element in a player's private information set is encoded as a prime number in the prime factorization of the market-clearing price. The element contained in every player's information set is identified as the prime with the highest multiplicity.

1 Introduction

In this short note, we describe an information aggregation mechanism that can be used by players before playing a game of strategic complementarities under incomplete information. In such a game, players may have an incentive to share overly optimistic information with other players, thus inducing them to play higher actions. In this mechanism, players trade a token before playing the game. Players who want to communicate good news must purchase this worthless token and burn resources. The note shows that players only need to observe the market clearing price that arises from the token trades to aggregate their private information. Each element in a player's private information set is encoded as a prime in the prime factorization of the market clearing price. The element that is contained in every player's information set is identified as the prime with the highest multiplicity.

For early-stage start-ups, incomplete information about the future returns of the start-up is an important friction in the fundraising process. Different investors have different beliefs about the quality of the start-up, different beliefs about how investors will evaluate the quality of the start-up, and possibly even different beliefs about those beliefs and so on.

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Moreover, if the future profits of the project exhibit some form of strategic complementarities, agents have the incentive to misreport what they know: If an agent believes the project to be just weakly above average quality, she may exaggerate her beliefs about the quality of the project so that other investors over-invest into the project. Indeed, over-investment of other agents weakly raises her returns. We see that information aggregation in this context requires a more complicated mechanism, where optimistic reports that are meant to increase the aggregate level of investment become costly. The simplest information aggregation mechanism is a public message board, where posting information is free and agents take turns reporting what they know. This cannot be used to incentivize information aggregation. In this note we propose a token trading protocol that achieve this.

1.1 Model

Consider a set I of $n \in \mathbb{N}$ agents, where each $i \in I$ is endowed with a cash endowment $b_i \in \mathbb{R}$ that enters her preferences in a quasi-linear way. Agents decide how much to invest into a project whose payoffs exhibit strategic complementarities. Returns of this investment game depend on an unknown quality parameter $\theta \in \Theta := \mathbb{Z}$. The investment game is described as follows: Each player chooses a level of investment $y_i \geq 0$. Given a profile of actions $y = (y_i)_i$ and a quality parameter $\theta \in \Theta$, agent i's payoffs are given by

$$U(y, \theta, b_i) = u(y, \theta) + b_i. \tag{1.1}$$

where u is smooth, strictly concave and supermodular with bounded derivatives. We assume that U is such that for every $\theta, (b_i)_i$, the complete information game with payoffs $U(\cdot)$ and actions $y = (y_i)_i$ has a unique pure strategy Nash equilibrium where at least one player invests a positive amount, denoted $y^*(\theta, (b_j)_j) = (y_i^*(\theta, (b_j)_j))_i$. We also assume that U is such that y^* is strictly increasing and strictly concave with bounded derivatives. In addition, zero investment is always assumed to be a Nash equilibrium. We will later require that these functions be sufficiently concave.

Information Players' private information is given by an improper, uniform prior ϕ on Θ and private signals $s = (s_i)_i$, where for every player i, the signal s_i is obtained as follows:

$$s_i = \theta + \epsilon_i, \tag{1.2}$$

where for every i, the noise term ϵ_i is drawn independently of θ and independently across players from a uniform distribution $\nu_i \in \Delta(\mathbb{Z})$ with finite support and mean zero. The tuple of $(\nu_i)_i$ thus describes a Bayesian game, where players' beliefs about θ are uniformly distributed on some interval. We denote the associated posterior beliefs of player i about θ , conditional on observing signal s_i by $\mu_i(\cdot|s_i)$. The improper distribution on tuples $(\theta, (s_i)_i)$ is denoted by μ.

We say that $(\nu_i)_i$ satisfy the *aggregate consensus* property if the following condition holds: For every $(\theta, (s_i)_i)$ in the support of μ,

$$\bigcap_{i \in I} \{\theta' \in \Theta : \mu_i(\theta|s_i) > 0\} = \{\theta\}. \tag{1.3}$$

Suppose the noise term satisfies $\epsilon_i \in \{-4, \ldots, 0, \ldots, 4\}$. With some abuse of notation, we will denote the first-order beliefs of agent i upon observing s_i with $\mu(\cdot|s_i)$. The distribution on signals s_i conditional on θ for the three agents is summarized below:

Signals given θ									
Signals:	$\theta-4$	$\theta-3$	$\theta-2$	$\theta-1$	θ	$\theta+1$	$\theta+2$	$\theta+3$	$\theta+4$
Signal distribution i_1:	0	0	0	$1/3$	$1/3$	$1/3$	0	0	0
Signal distribution i_2:	0	0	$1/3$	0	$1/3$	0	$1/3$	0	0
Signal distribution i_3:	$1/5$	$1/5$	0	0	$1/5$	0	0	$1/5$	$1/5$

Figure 1: Signal distributions for agents i_1, i_2 and i_3 conditional on the true state being θ.

So if agent i_1 observes signal s_1, she knows $\theta \in \{s_1 - 1, s_1, s_1 + 1\}$. She also knows that if $\theta = s_1 - 1$, say, then i_2' signal is contained in $\{s_1 - 3, s_1 - 1, s_1 + 1\}$. If $s_2 = s_1 - 1$, then i_2 knows that $\theta \in \{s_1 - 3, s_1 - 1, s_1 + 1\}$. In this case, agents i_1 and i_2 would not learn the state if they were to reveal the support of their beliefs. Both agents can only narrow the state down to either $s_1 - 1$ or $s_1 + 1$. Note that if $\theta = s_1 - 1$, then i_3 knows that $\theta \neq s_1 + 1$ no matter which of her signals she observed. So all three agents can learn the state if they manage to credibly communicate the support of their first-order beliefs. It can be verified that this is true for any state and any draw of the signals. So the beliefs described above satisfy the consensus condition.

Information Aggregation Mechanism Suppose that agents participate in a mechanism after they received their private signals but before they have to make their investment choices. The mechanism considered in this note is a special kind of mechanism. Suppose all agents are endowed with a large number of worthless tokens. An information aggregation mechanism consists of a token market. Every agent i reports a demand or supply schedule d_i for tokens $p \mapsto d_i(p)$, where $d_i(\cdot)$ takes as argument a positive price per token and maps it to a number of tokens. If the amount is positive, $d_i(p)$ represents a demand for tokens. If the amount is negative it represents a supply of tokens. For every profile of demand/supply schedules, let the public signal be given by a market clearing price[1], i.e. a number $p^* > 0$ so that

$$\sum_{i \in I} d_i(p^*) = 0. \tag{1.4}$$

At every round, agents effectively submit demand/supply schedules for tokens and observe the resulting market price of a token. Then agents pay or receive payment for tokens in units of endowment as dictated by their submitted demand/supply schedules and the market price. Hence transfers given $d = (d_i)_i, p$ take the form

$$\tau(d_i, d) := -p^* d_i(p^*). \tag{1.5}$$

[1]When multiple such prices exist we consider any selection rule defined on the aggregate schedules $\sum_{i \in I} d_i(\cdot)$. If no market clearing price exists, set p to zero.

Agents choose their reported demand/supply schedule d_i as a function of their private signal s_i. A strategy in the aggregation mechanism is then denoted $s_i \mapsto d_i(\cdot|s_i)$. We say that information has been aggregated if for every $(\theta, s = (s_i)_i)$ in the support of μ,

$$Pr(\theta|s_i, p^*) = 1, \ \forall \ i. \tag{1.6}$$

In the above expression, p^* is the value of the market clearing price and strategies are common knowledge. Note that no agent needs to condition on their own strategy $d_i(\cdot|s_i)$ to deduce the true state, the price and her private signal are enough.

Agents choose their strategy in the extended, two-period game, which consists of a strategy in the aggregation mechanism $s_i \mapsto d_i(\cdot|s_i)$ and an investment strategy $(s_i, p^*) \mapsto y_i(s_i, p^*) \geq 0$. Agent i's expected payoff before the trades, given a strategy $(d_i, y_i)_i$, is given by

$$U_i(s_i, y, d) := \mathbb{E}(u(y(s))\theta + b_i - p^* d_i(p^*|s_i)|s_i) \tag{1.7}$$

and expected payoffs after trade, but before the investment game:

$$U_i^*(s_i, y, d, p^*) := \mathbb{E}(u(y(s))\theta + b_i - p^* d_i(p^*|s_i)|s_i, p^*). \tag{1.8}$$

A Bayes-Nash equilibrium of this game consists of a strategy profile $(d_i, y_i)_i$ so that for every player i and every signal s_i, (d_i, y_i) maximizes U_i^* above given (d_{-i}, y_{-i}).

2 The Mechanism

We now provide an explicit strategy for every agent i which is a Bayes Nash equilibrium of the game and aggregates information. For every $m \in \mathbb{N}_{>0}$, let P_m denote the m-th prime number. That is $P_1 = 2$, $P_2 = 3$, $P_3 = 5$ etc... Fix any increaseing function $f : \mathbb{N} \to \mathbb{N}$ and let $\rho_f : \Theta \to \mathbb{N}$ be defined as follows

$$\rho_f(\theta) := \begin{cases} P_1, \text{ if } \theta \leq 1 \\ P_{f(\theta)}, \text{ otherwise.} \end{cases} \tag{2.1}$$

We then propose the following strategy $(d_i, y_i)_i$ for any given $\alpha > 0$:

(i) For every s_i, define the associated demand/supply schedule for player i under signal s_i

$$d_i(p|s_i) = \frac{\alpha n}{p} \sum_{\theta : \mu_i(\theta|s_i) > 0} \log(\rho_f(\theta)) - 1. \tag{2.2}$$

The market clearing price p^* associated to a signal profile $s = (s_i)_i$ satisfies

$$\begin{gathered} \frac{\alpha n}{p^*} \sum_{i \in I} \sum_{\theta : \mu_i(\theta|s_i) > 0} \log(\rho_f(\theta)) - n = 0 \\ \alpha \sum_{i \in I} \sum_{\theta : \mu_i(\theta|s_i) > 0} \log(\rho_f(\theta)) = p^* \end{gathered} \tag{2.3}$$

We now verify that $d = (d_i)_i$ aggregates information: Indeed, upon seeing p^*, each agent can perform the following computation:

$$\begin{aligned} \exp\left(\frac{p^*}{\alpha}\right) &= \exp\left(\sum_{i\in I}\sum_{\theta:\mu_i(\theta|s_i)>0}\log(\rho_f(\theta))\right) \\ &= \prod_{\theta:\exists\ i \text{ s.t. } \mu_i(\theta|s_i)>0} \rho_f(\theta)^{\eta(\theta)}, \end{aligned} \tag{2.4}$$

where for every $\theta \in \Theta$, $\eta(\theta)$ is the number of agents whose information set contains θ,

$$\eta(\theta) := |\{i \in I : \mu_i(\theta|s_i) > 0\}|. \tag{2.5}$$

Assuming that every agent can compute the prime factorization of $\exp(p^*)$, the aggregate consensus property implies that for any drawn tuple $(\theta^*, s = (s_i)_i)$ we have that

$$\eta(\theta^*) > \eta(\theta),\ \forall\ \theta \neq \theta^*. \tag{2.6}$$

Hence information is aggregated.

(ii) Let $d = (d_i)_i$, as defined above, be played during the information aggregation mechanism stage, and let the associated market clearing price p^* reveal state θ^*. We then consider an investment strategy $y = (y_i\colon (s_i, p^*) \mapsto y_i(s_i, p^*) = y_i(p^*))_i$, which is independent of each player's private signal s_i and let y be the Nash equilibrium $y^*(\theta^*, (b_i - p^* d_i(p^*))_i)$ of the complete information game with payoffs

$$U(y, \theta^*, b_i - p^* d_i(p^*)). \tag{2.7}$$

Moreover, we assume that if a prime factorization that is inconsistent with the information structure is observed, each agent invests zero.

We now check that no agent has an incentive to deviate during the information aggregation stage. Suppose agent i observes signal s_i. Reporting a higher information set means buying more tokens and may increase the consensus state that results from the market clearing price. Conversely, reporting a lower information is cheaper during the information aggregation stage but might result in a more pessimistic consensus.

A demand schedule $d_i'(\cdot)$ is more optimistic than $d_i(\cdot|s_i)$ if there exists $\Theta' \subseteq \Theta$ containing some $\theta > 2$, which is also greater then $\Theta(s_i) := \{\theta : \mu_i(\theta|s_i > 0\}$ in the weak set order: 1) For every $\theta \in \Theta(s_i)$ there exists $\theta' \in \Theta'$ so that $\theta \leq \theta'$ and 2) For every $\theta' \in \Theta'$ there exists $\theta \in \Theta(s_i)$ so that $\theta \leq \theta'$. Symmetrically, a demand schedule $d_i'(\cdot)$ is more pessimistic than $d_i(\cdot|s_i)$ if there exists $\Theta' \subseteq \Theta$, so that $\Theta(s_i)$ is greater than Θ' in the weak set order.

Suppose first that an agent reports a demand schedule $d_i'(\cdot)$ that is strictly more optimistic than the prescribed demand schedule $d_i(\cdot|s_i)$. The additional cost is bounded from below by

$$p'^* d_i'(p'^*) - p^* d_i(p^*|s_i) > c_f(s_i) := \alpha n \min_{\theta\in\Theta(s_i)} (\log(\rho_f(\theta+1)) - \log(\rho_f(\theta))), \tag{2.8}$$

where p^* is the price that results from the demand profile d and p'^* is the price resulting from player i's deviation. The Betrand-Chebyshev Theorem states that for any $l \in \mathbb{N}$ there is a prime number contained in $[l, 2l]$. Hence, we can find a strictly increasing function $f\colon \mathbb{N} \to \mathbb{N}$ so that for every $\theta \in \mathbb{N}$,

$$\log(\rho_f(\theta)) \in ((\theta - 1)\log(2), \theta \log(2)]. \tag{2.9}$$

Hence log primes can be used to span an approximately even grid. Hence, there is a choice of f so that

$$c_f(s_i) \geq \alpha n \log(2). \tag{2.10}$$

Let $\overline{\Theta}(s_i)$ denote the set of possible states that could arise as consensus with positive probability:

$$\overline{\Theta}(s_i) := \bigcup_{s_{-i}:\mu(s_i,s_{-i})>0} \bigcup_{j\neq i} \Theta_j(s_j). \tag{2.11}$$

The set $\overline{\Theta}(s_i)$ is finite and its cardinality is bounded uniformly by M for all signals s_i. Since agents invest zero if they see a prime factorization that is inconsistent with the information structure and information sets have finite size independent of the realized signals, no deviation encoding elements outside of $\overline{\Theta}(s_i)$ could ever be profitable. In the best case scenario, information aggregation after a deviation leads agents to the (possibly incorrect) consensus $\overline{\theta} := \max \overline{\Theta}(s_i)$. Let $\underline{\theta} := \min \Theta(s_i)$. Then the payoff gains from deviating to a more optimistic demand schedule under state θ^* is bounded from above by

$$\left(\max_{y_i \geq 0} u(y_i, y^*_{-i}(\overline{\theta}, (\overline{b}^*_j + \log(2))_{j\neq i}), \theta^*) - u(y^*(\underline{\theta}), (\underline{b}^*_j)_j, \theta^*) \right) - n\log(2), \tag{2.12}$$

where $\overline{b}^*_j$ and $\underline{b}^*_j$ are the highest and (resp.) lowest possible post-trade endowment for agent j given s_i if d is followed:

$$\begin{aligned} \overline{b}^*_j &:= \max_{s_{-i}:\mu(s_i,s_{-i})>0} (b_j - \tau(d_j(s_j), d_{-j}(s_{-j})). \\ \underline{b}^*_j &:= \min_{s_{-i}:\mu(s_i,s_{-i})>0} (b_j - \tau(d_j(s_j), d_{-j}(s_{-j})). \end{aligned} \tag{2.13}$$

By the bounds above we have that there exists M so that for all signals s_i,

$$\begin{aligned} |\overline{b}^*_j - \underline{b}^*_j| &\leq M, \\ |\overline{\theta} - \underline{\theta}| &\leq M. \end{aligned} \tag{2.14}$$

We will thus assume that u and y^* be sufficiently concave with bounded derivatives, so that there exists a constant $L \in (0, 1)$ so that for all endowments b all $\theta \in \Theta$

$$|u(y^*(\theta + M, (b_j + M + \log(2))_j), b_i, \theta + M) - u(y^*(\theta, b), b_i, \theta)| \leq Ln\log(2). \tag{2.15}$$

We conclude that a deviation towards a more optimistic demand schedule is never profitable. Similarly, a symmetric argument shows that deviating to a less optimistic demand schedule is not profitable. Finally, by concavity, no mixture of a optimistic and a pessimistic deviation would ever be optimal.

3 Discussion

3.1 Natural Encryption

Note that the equilibrium described above naturally encrypts the state whenever factoring a number into its basis elements is costly. For an outside observer without any private information about the state, the factorization can be made arbitrarily costly when using the protocol we introduced earlier by varying f. Agents who participate in the mechanism know which primes they have used in the encoding. Under a factorization cost, this mechanism seems more suitable for information aggregation among well informed experts. That is agents who already know a lot but also care a lot about aggregating their information without revealing their information to ignorant outsiders.

3.2 Related Literature

Information aggregation has been studied mostly in the context of auctions and competitive markets to provide micro-foundations for rational expectation equilibria Milgrom (1981); Reny and Perry (2006); Grossman and Stiglitz (1976); Siga and Mihm (2021). By contrast, in this paper, we view information aggregation from a mechanism design perspective. We want to design a mechanism whose sole purpose is to eliminate or reduce the friction of incomplete information when playing an investment game. Our mechanism should ideally be an add-on to investment projects in a broad class of different environments.

There is also a literature Kyle (1985); Vives (2014); Lambert et al. (2014) on information aggregation where traders submit monotone demand (or supply) schedules. This strand is close in spirit to our paper as it studies markets with strategic traders who receive private information and a market maker who determines prices. Like in our mechanism, trading is dynamic and information revelation occurs over time. In our mechanism, the message space is also given by demand/supply schedules. However, the asset being traded here has no intrinsic value.

Since we are focusing on supermodular investment games, we also relate to a larger literature on information frictions in coordination games. Examples like global games Carlsson and Van Damme (1993) and the Email game Rubinstein (1989) have shown that incomplete information can have dramatic consequences on the outcomes of such coordination games. Applications include currency attacks, and bank runs but also simple investment problems Morris and Shin (2003). Our mechanism can be viewed as a general tool to reduce these problems through mechanism design. However, we want those tools to find applications beyond the scope of two-player binary action supermodular games with global game information structures.

From this angle, a related paper is Angeletos and Werning (2006), where a financial market is used to prevent the global game selection that is introduced by perturbing private information of agents in a large economy. In their set-up, a public but noisy signal that results from the market price of an asset is used as a device to facilitate coordination - which is eliminated in a global games information structure. Their mechanism is tailored to the class of information structures that yield the global game selection, which applies mainly to two-player binary-action super-modular games[2]. In this paper, we take the role of information aggregation more literally and require the trading mechanism to yield a consensus. An important feature with many-action supermodular games is that agents have an incentive to exaggerate moderately good news: If agent i knows that the state is good but not great, revealing this to other agents only results in moderate investment levels. If instead, she could convince others that the state is great, this would incentivize other agents to provide much higher investment levels, which in turn benefits i. When there are only two actions, The state is either too low to invest (even if all other players invest) or it is high enough, in which case full investment is an equilibrium anyway.

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[2]While many interpretations of global games feature a continuum of states and players, there is usually a binary regime state which switches with a cut-off rule. This makes it effectively a two-player, binary action supermodular game with a continuum of types per player.

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