Automata representations of information with applications to information design

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Harvard / MIT Theory Seminar, Oct. 2022

Study of a game when information varies

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What outcomes?

What is the power of information? What outcomes can a designer implement by information dissemination?

What information?

Information design: How to implement outcomes

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Information design: How to implement outcomes

What model?
Revelation principle: Sufficient and minimal model of information

Benchmark: Correlated Equilibria

Aumann, 74, 87, Forges 93, Bergemann Morris 16

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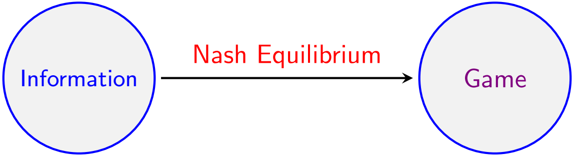

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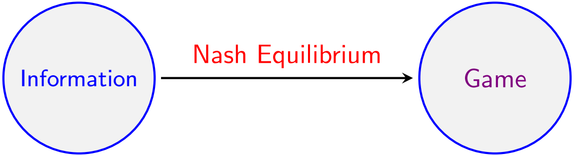

Correlated Equilibrium Distribution (ced): Distribution on game outcomes induced by some Nash equilibrium for some information

Revelation Principle

All ced are induced in the following way
1/ Information: each player is informed of a recommended action
2/ Nash equilibrium: each player plays the recommended action

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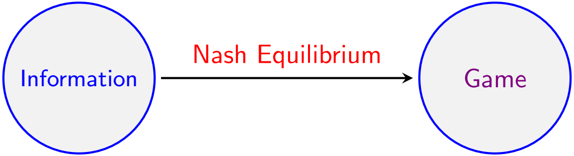

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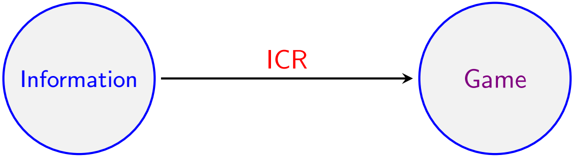
Characterizes what can be obtained and how to obtain it assuming the designer can choose 1/ information and 2/ the Nash Equilibrium

What can be implemented in all Nash equilibria?

Interim Correlated Rationalizablility (Dekel Fudenberg Morris 07): iterative deletion of dominated strategies



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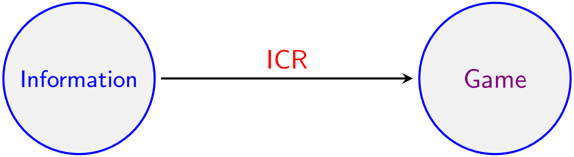

Rationalizable Distribution: Distribution on the actions of the game induced by ICR for some information

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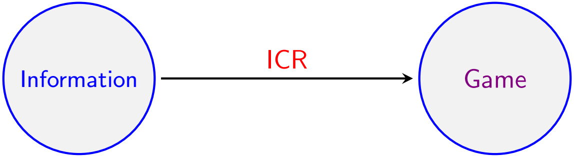

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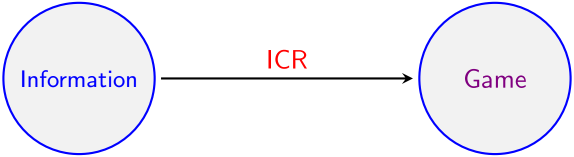

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Open problem for all games

Stationary Canonical Automaton Markov Priors (SCAMP)

Simple information structures implement all rationalizable distributions

1. Characterize and compute rationalizable distributions

2. Design information structures

Generalize email game and information structures in OT (21) MOT (22)

Plan

1. Interim Correlated Rationalizability

2. SCAMP

Interim Correlated Rationalizability

Game with incomplete info: $K, (A_i)_i, u_i : K \times A \to \mathbb{R}$, $B_i = \mathcal{P}(A_i)$

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Best-responses to $p_i \in \Delta(K \times B_{-i})$:

- Conjecture $\sigma(k, b_{-i}) \in \Delta(b_{-i})$ for every $k \in K, b_{-i} \in B_{-i}$
- $\mathrm{BR}_i(p_i) = \bigcup_\sigma \{\arg\max \mathrm{E}_{p_i,\sigma} u_i(k, \cdot, a_{-i})\} \in B_i$

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Common priors information structure $P \in \Delta(K \times T)$, $T = \prod_i T_i$

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Interim Correlated Rationalizability

$$
\begin{aligned}
\mathrm{ICR}_i^0(t_i) &= A_i \\
\mathrm{ICR}_i^{m+1}(t_i) &= \mathrm{BR}_i(P(k, \mathrm{ICR}_{-i}^m(t_{-i})|t_i))) \\
\mathrm{ICR}_i^\infty(t_i) &= \cap_m \mathrm{ICR}_i^m(t_i)
\end{aligned}
$$

Rationalizable distribution

Rationalizable distribution $\mu \in \Delta(K \times B)$ induced by P

$$\text{when} \quad (k, t) \sim P, \quad (k, \mathrm{ICR}^\infty(t)) \sim \mu$$

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To answer these, we need to understand the (recursive) structure of ICR

Email game

	a	b
a	1,1	-2,0
b	0,-2	0,0

$Good$

	a	b
a	-1,-1	-2,0
b	0,-2	0,0

Bad

Email game

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$Good$

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Bad

$$\mathrm{ICR}_i^0 = ab$$

$$\begin{cases} \mathrm{ICR}_i^{n+1} & = b \text{ if } P(\mathrm{ICR}_j^n{=}b|t^i) + 1 > 2P(\mathrm{ICR}_j^n{=}ab, \mathrm{G}|t^i) \\ \mathrm{ICR}_i^{n+1} & = ab \text{ otherwise} \end{cases}$$

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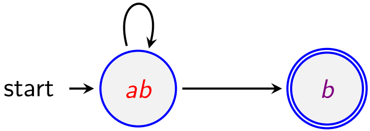

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Email game

	a	*b*
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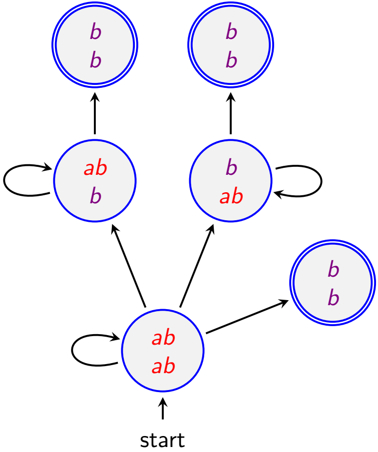

Good date / bad date (or technology adoption)

	a	b
a	1, 2	0, 0
b	0, 0	2, 1

G

	a	b
a	0, 1	0, 0
b	2, 2	1, 0

B

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$\mathrm{ICR}_1^1 = ab$ or b
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If $\text{ICR}_1^1 = ab$ then $\text{ICR}_1^2 = ab$ or a
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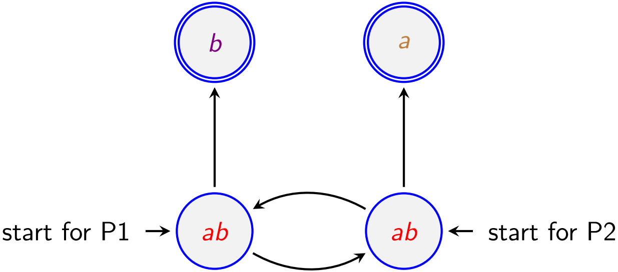

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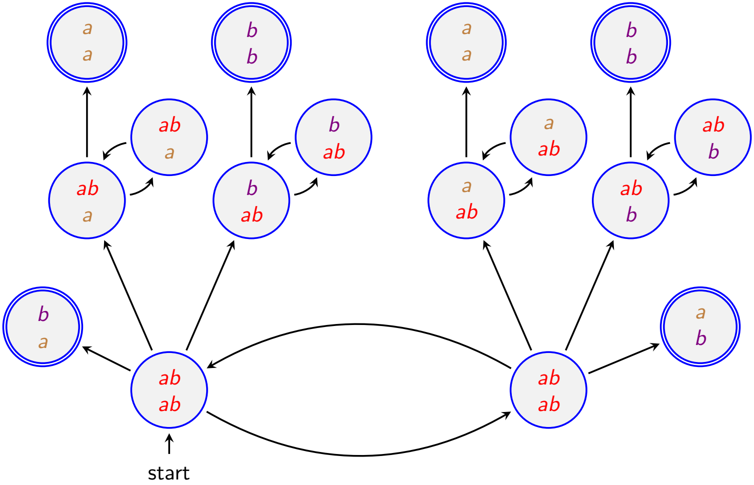

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Stationary Canonical Automaton Markov Prior

Automaton: finite set of states Ω, action maps $\beta_i \colon \Omega \to B_i$.

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Prior: $P \in \Delta(K \times \Omega^{\mathbb{N}})$, defines an information structure:

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Markov: $P \in \Delta(K \times \Omega^{\mathbb{N}})$ s.t.:

$$P(\omega^{n+1} | k, \omega^1 \dots \omega^n) = P(\omega^{n+1} | k, \omega^n) \quad P\ a.s.$$

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Player i's information $s_i = (s_i^n)_n$ is a sequence in $B_i^{\mathbb{N}}$

Canonical Prior P: s_i is i's ICR sequence itself, ie.

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Equivalent to the Obedience Constraints:

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A CAMP is both

- a distribution on nature and ICR hierarchies
- an information structure implementing this distribution

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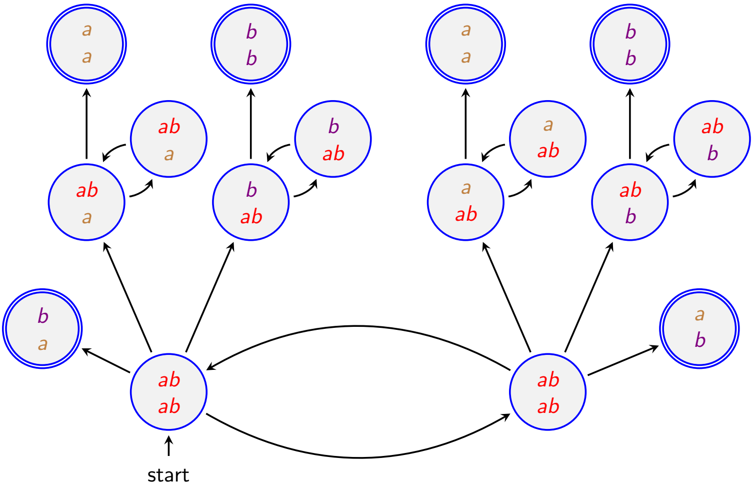

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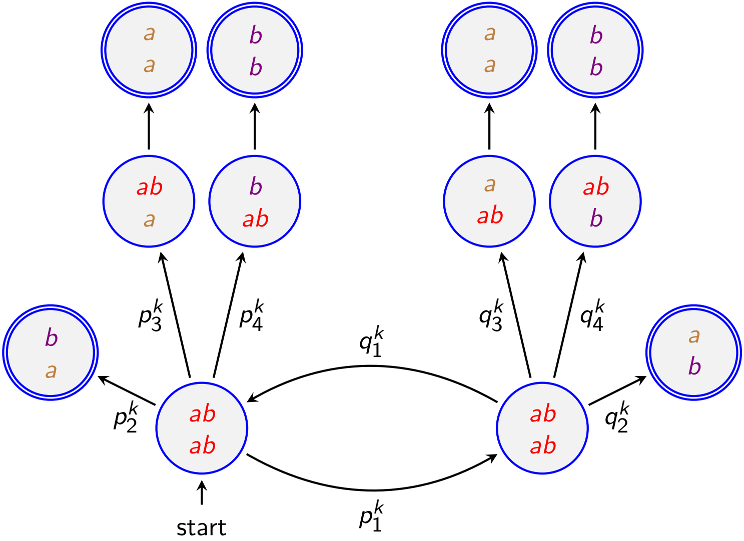

Stationary Canonical Automaton Markov Prior

Let m be the depth of the automaton, ie. the smallest value s.t. each state is reached with positive pba. Let c be the lcm of cycle lengths.

Stationarity: For every k there is a distribution P_k on terminal nodes st. for $\ell \geq 1$

$$P(\omega^{m+\ell c}{=}\omega | k, \omega^{m+\ell c} \text{ terminal}) = P_k(\omega)$$

In this case,

$$P(\omega^{\infty}{=}\omega | k) = P_k(\omega)$$

and

$$\mu(k, b) = P(k) P_k(\beta(\omega){=}b)$$

A revelation principle in SCAMP

Theorem

For every game, there exists an automaton st. rationalizable dist are

1. those on terminal nodes induced by all $P^m \in \Delta(K \times \Omega^m)$ that satisfy OC's
2. the distributions induced by SCAMP

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Convex polyhedron given by finitely many OC

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Information structures: SCAMP

- an information structure for every rat. dist.
- based on contagion, generalizes email and global games

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	a	b
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$p(G) = 1/2$

	a	b
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SCAMP gives a characterization of the set of rationalizable distributions, subset of $\Delta(K \times \{ab, a, b\}^2)$.

For visualization, focus on elements with support on $K \times \{a, b\}^2$.

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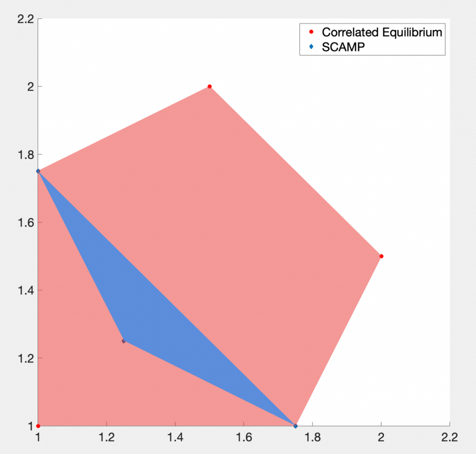

Extreme points

$\frac{1}{4}(G, a, a), \frac{1}{4}(G, b, b)$
$\frac{1}{4}(B, a, b), \frac{1}{4}(G, b, a)$

$\frac{1}{2}(G, a, a)$
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Conclusion

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ICR has a recursive structure on a proper automaton

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- Markov on Automaton finitely many parameters
- Stationary finitely many linear constraints
- Tractable and rich model of simple information

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Rationalizable distributions

- Closure is a convex polyhedron
- Are induced by SCAMP
- Generalization of contagion argument